DESIGN AND APPLICATION OF DDS-CONTROLLED, CARDIOID LOUDSPEAKER ARRAYS

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1 INTRODUCTION

After the introduction of Digital Directivity Synthesis (DDS) three years ago, AXYS DDS-driven arrays (like the Target and Intellivox-XL series) have been successfully applied during many music performances (front-of-house system) and also in a few fixed installs (PA and voice evacuation). Using DDS, which is based on a ‘constrained least squares’ optimization scheme, any desired 3D array response can be synthesized. Starting from a pre-defined array set-up and desired SPL distribution at the boundaries (including the audience area) of a (fictive) hall, the optimum output filters for the array elements (channels) can be calculated. Next, these output filters can be uploaded to all units in the array.

Using single line arrays, the front and the back lobe of the array cannot be controlled independently. The relative level of the backward radiated lobe is fully defined by the front-to-back ratio of the array elements and the shape of the array. To avoid unwanted reflections caused by the backward radiated lobe, it is recommended to mount the array on a large baffle (e.g. wall). In case of an electronically steered array the back lobe will be reflected in the same direction as the front lobe and will even enhance the front lobe (especially for low frequencies).

In practice however, mounting on a wall (or flush-mounting) is not always possible. For example in a time-aligned multiple-array set-up (e.g., along train platforms and in large airport terminals) free-standing arrays are often inevitable. Also, in indoor concert applications freestanding or flown (bass) arrays are very common. Ideally, the arrays in these situations should be unidirectional. Note that for higher frequencies this desired behaviour is closely approximated as the backward radiated lobe is relatively weak. However, for lower frequencies the arrays become almost omnidirectional in the horizontal plane.

In this context it is remarkable that in sound recording cardioid microphones are used very often to reject incoming sounds from the rear of the microphone. In sound reproduction however, cardioid loudspeakers are less common. One example is the Philips-Bosch cardioid column loudspeaker (LBC 3051-3053 range). These speakers make use of ‘acoustic filters’ in the form of slits in the outer part of the enclosure.

In this paper the design, optimisation and testing of an active, DDS-controlled, cardioid Intellivox loudspeaker array is presented. The array is driven in such a way that a strong directional behaviour in the vertical plane and a cardioid-like behaviour in the horizontal plane is obtained over a large frequency range.

The proposed optimisation technique can be applied to other array types as well (e.g. bass arrays). Due to their ‘unidirectional’ behaviour these cardioid loudspeaker arrays are expected to have many acoustic benefits; improved indoor bass response, higher direct-to-reverberant ratio, higher gain-before-feedback, improved echo-reduction in delayed set-ups, etc.

2 CARDIOID LOUDSPEAKERS

2.1 Theory

In theory, a cardioid loudspeaker can simply be made with two opposite polarity monopole sources separated by a distance $\Delta l$. The signal to one of the sources should be delayed by a time $\Delta l / c$, where $c$ is the speed of sound, as shown in Fig 1. The sound field of this cardioid is given by
\[ P(r_0, \theta) = A_{\text{front}}(f) e^{-jkr_1} + A_{\text{back}}(f) e^{-jkr_2} \]  
\[ (1) \]

where \( r_1 \) and \( r_2 \) are the distances from the monopole loudspeakers to a far field receiver position and \( k \) the wave number \((2\pi f/c)\). The complex factors \( A_{\text{front}} \) and \( A_{\text{back}} \) are given by

\[ A_{\text{front}}(f) = e^{j\beta \Delta l/2} \]  
\[ (2a) \]
\[ A_{\text{back}}(f) = -e^{-j\beta \Delta l/2} \]  
\[ (2b) \]

with \( \beta = 1 \) in this situation. In general, \( \beta \) can be used to change the phase correction.

![Diagram of cardioid source made by two closely spaced monopole loudspeakers with intermediate distance \( \Delta l \), processed with complex weighting factors \( A_{\text{front}} \) and \( A_{\text{back}} \).](image)

For small loudspeaker distances compared to the wavelength \((k\Delta l \ll 1)\) the sound field of the cardioid source can be approximated in the far field \((r_0 \gg \Delta l)\) by

\[ P(r_0, \theta) = jk\Delta l(\beta + \cos \theta) e^{-jkr_0} \]  
\[ (3) \]

where \( r_0 \) is the distance from the centre between the two loudspeakers to a far field position at angle \( \theta \). In this expression the directivity \( D \) is given by

\[ D(\theta) = \frac{(\beta + \cos \theta)}{\beta + 1} \]  
\[ (4) \]

Note that the cardioid frequency response is proportional with the intermediate distance \( \Delta l \) and with the wave number \( k \) (i.e. 6 dB increase per frequency doubling). Consequently, for small values of \( k\Delta l \) the cardioid source becomes very inefficient.

In the case of axial symmetric sources the directivity factor \( Q \) is given by

\[ Q = \frac{2}{\pi} \int_0^{\pi} D^2(\theta) \sin \theta d\theta \]  
\[ (5) \]

From Eq. 4 and 5 we find that
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\[ Q = \frac{(\beta + 1)^2}{\beta^2 + 1/3} \]  

(6)

Using Eq. 6, we find that \( Q = 3 \) for the cardioid source (\( \beta = 1 \)). Inserting \( \beta = 0 \) or \( \beta = 1/3 \) in Eq. 4 the well-known dipole (figure of eight) and hypercardioid characteristics are obtained, respectively. It can be shown by differentiating Eq. 6 that for \( \beta = 1/3 \) (i.e., hypercardioid source) the directivity factor is maximum. In Table 1 an overview is given of some standard directivity patterns including their directivity factor \( Q \) and directivity index \( DI \) (10 log\( Q \)).

<table>
<thead>
<tr>
<th>Pattern</th>
<th>( \beta )</th>
<th>( Q )</th>
<th>( DI ) [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopole</td>
<td>-</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Dipole</td>
<td>0</td>
<td>3</td>
<td>4.8</td>
</tr>
<tr>
<td>Cardioid</td>
<td>1</td>
<td>3</td>
<td>4.8</td>
</tr>
<tr>
<td>Hypercardioid</td>
<td>1/3</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

2.2 Simulations

To illustrate the formulas above, first a cardioid source (\( \beta = 1 \)) with an intermediate loudspeaker distance of 0.17 m is simulated. Fig. 2 shows the exactly calculated (using Eq. 1) and the approximated (using Eq. 3) polar diagrams for frequencies of 125, 250, 500, and 1k Hz. As expected, for increasing frequencies (i.e., increasing values of \( k\Delta l \)) the approximation of Eq. 3 becomes less accurate.

Fig 2: Polar diagrams of a cardioid source with intermediate speaker distance of 0.17 m for frequencies of 125, 250, 500 and 1k Hz (total range 30 dB).
Secondly, the response of a dipole ($\beta=0$) and hypercardioid ($\beta=1/3$) source is simulated. The polar diagrams for $f=250$ Hz are shown in Fig. 3.

![Polar diagrams for dipole and hypercardioid sources](image)

Fig 3: Far field polar diagrams of a dipole (left) and hypercardioid source (right) with intermediate loudspeaker distance of 0.17 m at 250 Hz (total range 30 dB).

2.3 Practical complications

Although the basic principles of cardioid loudspeakers are quite simple, the practical design is a bit more complicated. Most real loudspeakers aren’t perfect monopoles, even at low frequencies. Both the amplitude and the phase responses may vary with angle. Also, a slight imbalance of the two loudspeakers can ruin a perfect cancelling to rear direction of the cardioid. To make it even more complicated, the presence of one speaker may change the response of the other (mutual radiation impedance).

As described in paragraph 2.1, a careful balance must be found between the sensitivity at low frequencies and the desired cardioid behaviour at mid frequencies by choosing the optimum intermediate loudspeaker distance $\Delta l$.

In order to design a practical cardioid source, a detailed complex (amplitude and phase) directivity model must be available for both loudspeakers, each preferably measured in the presence of the other one. Based on these data, complex filters $A_{\text{front}}(f)$ and $A_{\text{back}}(f)$ can be derived for the front and the rear loudspeaker respectively.

3 DDS-OPTIMIZED CARDIOID-LIKE ARRAYS

3.1 Procedure

In order to combine the effect of a vertical line array (high vertical directivity) and the large front to back ratio of a cardioid source, a cardioid Intellivox loudspeaker array was built and tested. Such an array is expected to show a strong directional behaviour in the vertical plane and a cardioid-like behaviour in the horizontal plane for low and mid frequencies.

Using the DDA (Digital Directivity Analysis) software, it is fairly easy to optimise output filters for a cardioid array. First, a suitable array set-up was defined. Secondly, appropriate values for the desired response at the boundaries of a fictive hall were defined. Starting from this pre-defined array set-up, the optimum output filters for the array elements were calculated by the DDS-algorithm. Next, the chosen array configuration and the synthesis (i.e. optimisation) procedure will be described.
3.2 Cardioid array set-up

The array, which was used during the test, consisted of two Intellivox-2c-XL columns (acoustic length approx. 2 m), assembled back-to-back. Each column has 16 loudspeakers, driven with 8 channels. In contrast to the standard DDC Intellivox-2c columns, this XL-version can be DDS-optimised. The distance between the fronts of the cabinets was 150 mm.

3.3 Optimisation geometry

The (fictive) geometry, for which the array was optimised, is shown in Fig. 4. The geometry is determined by the floor, rear and front wall, and ceiling. The lowest loudspeaker in the array was positioned at 3 m above the floor. The audience plane was defined at 1.6 m above the floor.

Fig 4: Simulation geometry for the optimisation of the double Intellivox-2c-XL array.

The desired SPLs at the boundaries of the model were chosen as follows:

- Floor front: 90 dB.
- Floor back: $-\infty$ dB.
- Ceiling and end walls: $-\infty$ dB.

3.4 Calculation of the output filters

Using the DDS-algorithm in DDA, the desired array response is synthesized in the following way. Starting from the pre-defined array set-up and the desired SPL distribution, the optimum output filters are calculated for each channel in the test array. Next, these filters are exported to a dda-file in which all FIR coefficients and delays are included. These coefficients are uploaded to the DSPs through a network connection between a PC and the Intellivox units.
3.5 Simulated response of the cardioid array

In DDA the realised SPL distribution of the optimised array can be calculated. As an example the response at 250 Hz is shown in Fig. 5.

The simulation results in Fig. 5 show that this cardioid array exhibits a strong directional behaviour in the vertical plane and a cardioid-like behaviour in the horizontal plane. With DDA it can be verified that using the cardioid set-up the maximum continuous SPL in the audience plane for low frequencies is approx. 5 dB higher than for the single array. Note that adding a coherent array would result in a 6 dB increase. This means that the sensitivity of the total array is very well controlled.
4 MEASUREMENTS

4.1 Set-up

In order to verify the simulated response, outdoor measurements were done on the cardioid test array. The array was vertically flown into a scaffold positioned on a grass field. The lowest loudspeaker was raised to a height of 3 m. A number of 19 measurement positions were defined along a semi-circle with a radius of 15 m at 10-degree intervals from the front to the back of the array. At each receiver position a TDS measurement was done using a TEF20 system. The height of the microphone was 1.6 m. The time window was chosen such that reflections from buildings in the vicinity were eliminated. However, in order to maintain a sufficiently high frequency resolution (25 Hz), this time window was still too long to eliminate the ground reflection. It was assumed that for low and mid frequencies the ground reflection had a similar effect to the measurement results at all angles. Therefore, no correction was necessary.

First, the double Intellivox array was driven as a cardioid array. Secondly, for comparison, the array was uploaded with a single array setting (only one Intellivox active).

Since the measurements were taken outside, the noise conditions were quite poor for low frequencies (below 250 Hz). Especially at the backside of the cardioid, the results were affected by the background noise.

4.2 Results

The 1/3-octave averaged measurement results are shown in Fig. 6a-b for the single as well as the cardioid array set-up. For visualisation reasons, the results from 0-180° are mirrored to 180-360°.

Using DDA, the results are also simulated at the same positions as during the measurements.

Fig. 6a: Measured and simulated 1/3-octave ‘polar’ diagrams (100-315 Hz) for the single and the cardioid array set-up (10 dB/div).
Fig. 6b: Measured and simulated 1/3-octave ‘polar’ diagrams (400-5000 Hz) for the single and the cardioid array set-up (10 dB/div).
The results in Fig. 6a and 6b show that the cardioid-optimised array has a strong rejection to the rear over a large frequency range. For low and mid frequencies (below 630 Hz), an extra (i.e., compared to the single array) reduction of up to 20 dB can be realised. The maximum reduction is not always found at 180°, but shifts towards the sides (tendency to hypercardioid behaviour).

There is a good overall agreement between measured and simulated data. Due to the poor LF signal-to-noise ratio at the back, the simulated attenuation at those angles couldn’t be verified. For higher frequencies (above 630 Hz) the horizontal directivity pattern of the array is mainly determined by the directivity of the single array.

5 CONCLUSIONS

The AXYS, DDS technology has been applied to optimise an active cardioid loudspeaker array consisting of two separate Intellivox-2c-XL columns. The tested array set-up shows a strong directional behaviour in the vertical plane and a cardioid-like behaviour in the horizontal plane over a large frequency range. A backward rejection up to 20 dB can be realised.

The proposed approach has the following benefits:

- Generic concept. The DDS technique can be applied to arbitrary array configurations (e.g., bass arrays).
- Modular array set-up. A single array can be ‘upgraded’ if necessary to a cardioid array by adding a unit to the back.
- Radiation pattern can be fully customised by software (DDA).
- Automatic sensitivity optimisation. In the DDS algorithm the sensitivity is optimised in combination with the radiation pattern.

Future research will be focussed on a more efficient physical implementation of the cardioid array. Since the loudspeakers in the rear of the array are mainly active at low and mid frequencies, it is expected that a sparser loudspeaker distribution can be used without degrading the overall performance.

6 REFERENCES

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