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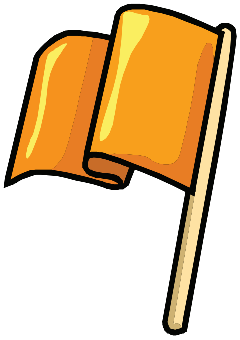

What is Waving and Why?

The Anatomy of the Wave Equation - Part 4

At the conclusion of the third article in this series we had learned that the acoustic energy at any instant contained in a one wavelength interval along the z-axis of the plane wave tube was 4.1288(10^{-9}) Joule. We also had learned that the oscillating piston supplied this amount of acoustic energy during one period of the piston's motion of 0.001 second. Now if an observer is positioned at any fixed value of the z-coordinate in the plane wave tube, this same amount of acoustical energy will pass the observation point while being transported in the positive z direction in a time of 0.001 second so the average acoustical power over this interval of time is W/T where W is the acoustical energy and T is the period of the sinusoidal piston motion. < *P* > denotes this average power and in this instance has the value 4.1288(10^{-6}) Watt. The average acoustical intensity denoted by < I > is a vector quantity defined to be the average directed power flow per unit area. As it is a vector quantity < I > has both a magnitude and direction. For a fixed location the magnitude of < I > amounts to the total acoustical energy flow averaged over the time of the flow per unit of area through which the energy passes. In this instance the magnitude of < I > = < *P* > /S where S is the cross-sectional area of the tube. In the present case the magnitude of < I > is 8.1482(10^{-3}) Watt•m^{-2} and the direction of < I > is the same as that of the wave propagation, namely the positive z-direction. The instantaneous intensity I(z, t) is a related physical quantity that is also a vector quantity. I(z, t) is a function of both position and time and is a measure of the instantaneous power flow per unit area at a particular location z and time t. It is calculated from the product of the acoustic pressure with the particle velocity at the z-coordinate and time coordinate of interest so I(z, t) = p(z, t)•u(z, t). For a plane wave propagating in the direction of increasing z, u(z, t) = p(z, t)/ρ_0c. For the case at hand, the direction of I(z, t) is always that of the positive z-axis. It is true that the particle velocity alternates between the positive and negative z direction, but the acoustic pressure is in phase with the particle velocity so that when the particle velocity is instantaneously in the negative z direction the acoustic pressure is negative and the overall product remains positive. Now < I > at some fixed point z can be calculated from the time average of I(z, t) at the same value of z and hence < I > = < $[p(z, t)]^2$ > /ρ_0c. The quantity < $[p(z, t)]^2$ > by definition is just the mean value of the square of the acoustic pressure so it is possible to write equation (1) as

$$\langle I \rangle = \frac{p_{rms}^2}{\rho_0 c}$$

This is a very useful equation. Even though it was derived by considering only a plane wave it is equally valid for a spherical wave.

It is probably safe to say that the most often made

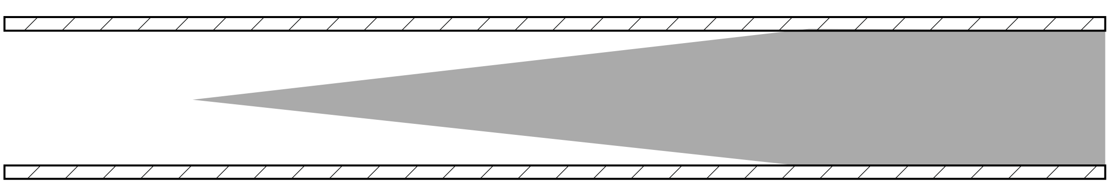

Figure 1. Possible plane wave tube structure.

measurement in acoustics is that of sound pressure level. Levels are logarithmic comparisons of a power or power-like quantity with regard to some standard reference value for the quantity in question. When the quantity is acoustic pressure, the power-like quantity is p^2_{rms} and the reference is $[20(10^{-6})]^2$ Pascal^2. Strictly speaking, the sound pressure level in decibels would be written as equation (2) with the form

$$\mathrm{SPL} = 10\log_{10}\left(\frac{p_{rms}}{20(10^{-6})}\right)^2 \mathrm{dB} = 20\log_{10}\left(\frac{p_{rms}}{20(10^{-6})}\right)\mathrm{dB}$$

In the third article in this series we learned that the root mean square acoustic pressure for the sound wave in this plane wave tube has a value of 1.834 Pascal. When this value is substituted into equation (2), the calculated sound pressure level becomes 99.25 dB. Additionally, when p_{rms} is substituted into equation (1) with $\rho_O c$ having a value matching the ambient conditions, namely 412.8 Rayls, the magnitude of $< I >$ is found to be $8.1482(10^{-3})$ Watt•m^{-2}. Now $< I >$ is a true power-like quantity and the reference value for average acoustic intensity is 10^{-12} Watt•m^{-2}. The intensity level or IL is then calculated from equation (3), which is

$$\mathrm{IL} = 10\log_{10}\left(\frac{< I >}{10^{-12}}\right) = 99.1\mathrm{dB}$$

There are two reasons for the small discrepancy in a given physical situation between the numerical values for SPL and IL. Firstly, the reference values, although close, are not exactly equivalent and the specific acoustic impedance of air for plane waves varies dependent upon the ambient total atmospheric pressure and the absolute temperature. For the range of ambient conditions usually encountered in practice they will agree within a fraction of a decibel as was the case in our example. For all practical purposes, then, the SPL and the IL can be taken as one and the same.

Plane wave tubes are often used for measuring the properties of transducers in general and high frequency compression drivers in particular. We have seen that a uniformly constructed tube of infinite length with excitation at one end allows the existence of a single traveling plane wave in just one direction. Such a device is obviously not physically possible. We need to visualize a device of finite length that maintains uniform geometry and does not produce reflections because of its finite length. Two such possibilities are suggested in Figs. 1 and 2.

Figs. 1 and 2 are cross-sectional drawings of viable plane wave tube structures. In each instance the inner tube diameter is greatly exaggerated as compared with the actual tube length. Typical inner diameters for use with compression drivers must exactly match the exit apertures of the drivers of interest. The inner diameters would then be of the order of an inch or so but the tube must be ten or more feet in length. In both instances the structure is a figure of revolution about the central axis so that cylindrical symmetry is maintained over the entire length of the structure. As a result there are no abrupt changes in geometry. The interior shaded regions in both instances are occupied by a uniform open cell acoustical foam that has a specific acoustical impedance that matches as closely as possible that of air in the range of 410 to 415 Rayls. The tubes must be mounted vertically because acoustical foam is flexible and a horizontal mount would lead to sag of the foam that would destroy the cylindrical symmetry. This would be particularly true for the conical foam structure of Fig. 1. The hollow tube employed in the structure in both instances must have smooth interior walls of sufficient thickness to be rigid.

The rationale behind the proposed structure is quite straightforward. Having an exact match between driver exit diameter and the tube's inner diameter ensures two things. In the first instance the driver's compression ratio will not be influenced by its attachment to the tube and secondly, there will not be an abrupt change in geometry that would cause a reflection back into the driver. In the air-filled space near the driver the plane wave propagates in the normal fashion and as the wave progresses down the tube it encounters no abrupt geometry

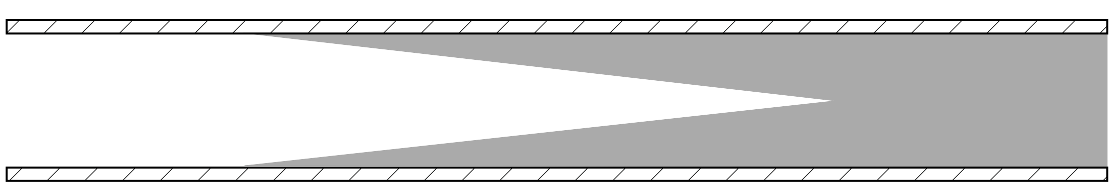

Figure 2. Alternative plane wave tube structure.

changes and is always in media having a common value of specific acoustical impedance. The portion of the wave in the free air is undergoing an adiabatic process and loses no acoustical energy. The portion of the wave in the acoustical foam on the other hand is in a medium of much higher thermal conductivity and is undergoing a predominantly isothermal process where acoustical energy is being dissipated as heat. Finally the greatly attenuated wave enters a region entirely filled with foam for a sufficient length that the remaining acoustical energy for all practical purposes is completely absorbed by the time the end of the tube is reached. In summary, there are no reflections back towards the source. The downside associated with these structures is the necessity of the vertical orientation, where ceiling heights are restricted, and the expense involved in accurately shaping the acoustical foam.

As an alternative, Fig. 3 depicts what might be called a poor man's plane wave tube.

In viewing Fig. 3 the reader should be aware that the dimensional scales associated with the driver mounting flange, the microphone port, and the tube diameter are greatly magnified relative to the tube length in order to show construction details. The microphone port should be as close to the driver as possible and should make a tight fit with the body of a pressure sensitive microphone assumed to be cylindrical in shape. The microphone should be no more than 0.5 inch in diameter with a 0.25-inch diameter microphone being preferred. In either case the microphone capsule's protective grid should be removed and the microphone should be positioned such that its diaphragm's surface is just tangent with the inner wall of the plane wave tube. Half of the length of the tube is to be filled with graduated stuffing of ordinary fiberglass building insulation. The shaded interior portion of the drawing in Fig. 3 indicates this. The graduated stuffing is prepared in the following way. The final one-foot length near the end of the tube should be compacted firmly and then the amount of compaction should be gradually reduced until the midpoint of the tube is reached. Thick-walled PVC piping of the appropriate inner diameter may be employed for the plane wave tube and a smaller pipe with a plugged end with incremental length markings can be employed as a stuffing plunger while working with small tuffs of the fiberglass. The principal tube may be mounted horizontally if it is provided with adequate supports to keep the tube level. Several squares of 3/4 inch plywood with an appropriate center hole sized to fit the o.d. of the plane wave tube are adequate. The graded absorber is admittedly not perfect. If care is taken in making it, however, any reflections will be close to 40 dB down. If the distance between the microphone port and the face of the graded absorber is at least 5 feet then the first possible reflection will return to the measuring port after a time of 10 feet / 1128 feet per second or approximately 0.009 second. Furthermore, if one employs a TEF, Sysid, or similar measurement program the contribution from any reflections can be screened out with only a moderate loss in the frequency resolution of the measurement. If sufficient horizontal space is available, a tube of twenty feet of overall length with ten feet of graduated stuffing will allow accurate measurement to as low as 50 Hz.

There is one caution related to high frequency operation. A cylindrical wave guide can support modes of wave propagation other than that of plane waves if the guide is suitably excited above a frequency as given by equation (4),

$$f = \frac{1.84c}{\pi d}$$

where d is the inner diameter of the wave-guide. This frequency is approximately 8 kHz for a one-inch diameter guide and of course 4 kHz for a two-inch diameter guide. This matter as well as wave-guide behavior for various impedance miss-matches will be covered in a forthcoming article. *ep*

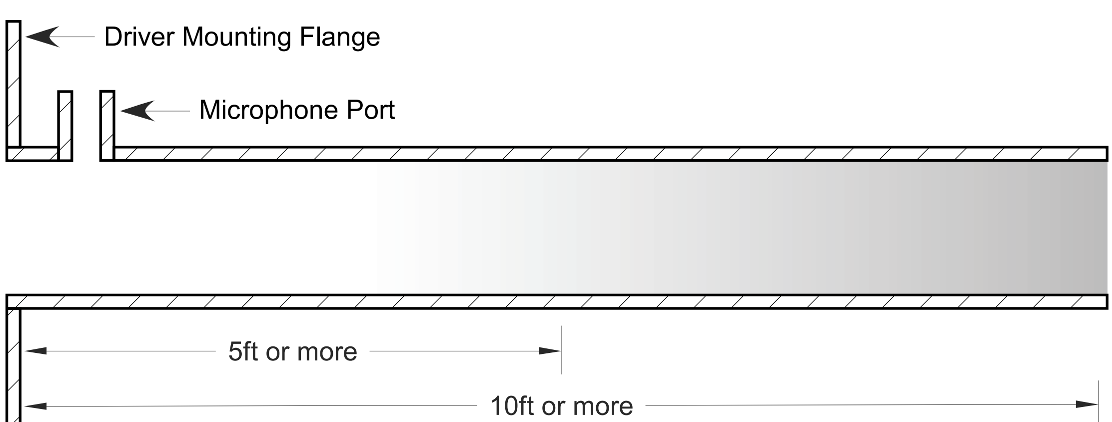

Figure 3. Poor man's plane wave tube for horizontal mounting.